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Analytical calculations of secular perturbations of translational-rotational motion of a non-stationary triaxial body in the central field of attraction

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The translational-rotational motion of two non-stationary bodies – a spherical body and a triaxial body is investigated. It is assumed that the initial dynamic shapes of bodies are preserved but their masses and sizes change in time [1], [2]. Besides, the reactive forces and additional torques are assumed to be small and may be neglected. An approximate expression for the force function of the Newtonian interaction accurate up to the second zonal harmonics is accepted. The translational-rotational motion of a triaxial non-stationary body is considered in a relative coordinate system with an origin situated in the center of a non-stationary spherical body. The axes of the own coordinate system of the non-stationary triaxial body are directed along its principle axes of inertia and we assume that in the course of evolution their relative orientation remains unchanged. Rotational motion is described in terms of the Euler variables. The problem is complex because the differential equations of motion are non-autonomous and have no integral. Therefore, the problem is investigated in the framework of the perturbation theory. Equations of motion in osculating analogues of Delaunay-Andoyer elements are derived in [1-5]. The unperturbed translational motion is described by an aperiodic motion on quasiconic section [1]. Unperturbed rotational motion is characterized by the Eulerian motion of non-stationary axisymmetric body [1], [2], [5]. Differential equations of the unperturbed translational-rotational motion are integrated by the Hamilton-Jacobi method. Differential equations of unperturbed translational-rotational motion of a non-stationary triaxial body were derived in Jacobi osculating variables. Equations of perturbed motion in the analogues of Delaunay-Andoyer elements have the canonical form

$$L = \frac{\partial F}{\partial l},\ \ G = \frac{\partial F}{\partial g},\ \ H = \frac{\partial F}{\partial h},\ \ l = -\frac{\partial F}{\partial L},\ \ g = -\frac{\partial F}{\partial G},\ \ h = -\frac{\partial F}{\partial H} \tag{1}$$

$$L' = \frac{\partial F'}{\partial l'},\ G' = \frac{\partial F'}{\partial g'},\ H' = \frac{\partial F'}{\partial h'},\ l' = -\frac{\partial F'}{\partial L'},\ g' = -\frac{\partial F'}{\partial G'},\ h' = -\frac{\partial F'}{\partial H'}. \tag{2}$$

The perturbing functions$F$, F' in (1), (2) written in the analogues of Delaunay-Andoyer elements are very complicated and one has to do a lot of symbolic computation to obtain them. Such computation can be performed efficiently with the aid of computer algebra systems. Finally, we obtain analytical expressions for the perturbing functions F, F' in the form

$$F = \frac{1}{\nu^2}\frac{\mu_0^2}{2\mu_0 L^2} + \left\{-\frac{1}{2}bR^2 + \frac{(m_1+m_2)}{m_1 m_2}U_2\right\},$$

$$F' = \frac{1}{2}\left(-\frac{1}{m\chi^2}\left[\frac{G'^2}{A_0} + \frac{A_0 - C_0}{A_0 C_0}L'^2\right]\right) - H_{1pert}^{rot} \qquad (3)$$

$$H_{1pert}^{rot} = \frac{1}{2}\left(\frac{B-A}{A^2}\right)\left(G'^2 - L'^2\right)\cos^2 l' - \left\{U_2 - \frac{1}{2}bR^2\right\} \qquad (4)$$

$$U_2 = fm_1\frac{A+B+C-3I}{2R^3}, I = A\alpha^2 + B\beta^2 + C\gamma^2, \qquad (5)$$

where f is the gravitational constant, the mass of non-stationary spherical body $m_1 = m_1(t)$ is a given function of time, A, B, C are the principle moments of inertia of non-stationary triaxial body, $A = A(t_0)\nu\chi^2$, $B = B(t_0)\nu\chi^2$, $C = C(t_0)\nu\chi^2$, $\nu = \nu(t)$, $\chi = \chi(t)$ are known dimensionless function of time, I is the moment of inertia of the non-stationary triaxial body relative to the axis given by the vector $\overrightarrow{O_1O_2} = \vec{R}$ connecting centers of mass of two bodies, $\alpha,\ \beta,\ \gamma$ are cosines of the angles formed by a straight line O_1O_2 with central axes of inertia of the non-stationary triaxial body. The perturbing functions F, F' (see (3)-(5)) are calculated analytically in terms of the Delaunay-Andoyer elements for the first time and may be obtained, in principle, with arbitrary accuracy. The corresponding complete expressions are very cumbersome and we do not show them here. Note that all time-consuming cumbersome analytical calculations are performed with the aid of the computer algebra system Mathematica [6], which has a convenient interface and makes it easy to combine different types of calculations. Further development of this work involves the study of the obtained equations for secular perturbations of translational-rotational motion of a triaxial body of constant dynamic shape and variable size and mass, using various analytical and numerical methods.

Keywords
Translational-rotational motion, Non-stationary triaxial body, Secular perturbations.

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