New Gronwall Type Inequality For the Caputo Fractional Differential Operator and Applications

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Time-fractional differential equations have been attractive in the past decades since many natural phenomena in physics, biology and chemistry can be described more precisely in this way. Numerous effort has been devoted in developing effective methods for time-fractional differential equations and simulations on a large range of physical problems. However, numerical analysis for time-fractional differential equations has not been well done, mainly due to the lack of a fundamental Gronwall type inequality. Such an inequality for first-order derivative and its approximations services as an essential tool in analysis of ODEs and PDEs. In this talk, we shall present our recent work in establishing a new fundamental algebraic Gronwall type inequality for several approximations to the Caputo fractional derivative, in terms of Mittag-Leffler function. Matlab software has also been used to verify our formulations. With the proved Gronwall type inequality, we provide theoretical analysis for several discrete algebraic methods. The theoretical results are illustrated by applying our proposed methods to three examples: linear Fokker-Planck equation, nonlinear Huxley equation and Fisher equation.