Modelling and Simulation of Solid Particle Sidementation in an Incompressible Newtonian Fluid.

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1 Introduction

The understanding of the physical phenomena that govern fluid / particle flows is continuously improving, particularly in the last decade. Transport phenomena and solid particles deposit in the context of hydraulic turbine engine systems is multidisciplinary. For modeling and simulation of such flows, there are several methods which use dynamic meshing. These methods follows the movement of the objects in a Lagrangian way [1, 2, 3]. However, the remeshing steps can be expensive and very difficult, especially in the 3D case.

To overcome the constraints caused by the use of adaptive meshing and reduce the problems associated with linking steps, new methods with fixed meshing are used. These are also called methods of fictitious domain as they extend a problem defined on a mobile and complex area (the fluid domain) to a domain (fictional) larger but fixed. **R. Glowinski et al** (see [4, 5, 6, 7, 8, 9]) are the investigators of fictitious domain methods. If the fixed field is sufficiently simple, this kind of method allows the use of Cartesian meshes, which allows the use of fast solvers.

Although the Navier-Stokes equations describing the behavior of a fluid still admit no evidence of existence of a general solution, they are still widely used to describe Newtonian fluid flows. In the case of the presence of particles in the fluid, the processing of the interaction between the fluid phase and the solid phase adds complexity to the studied problem.

In this paper we present a method of simulating the movement of one or more convex rigid body in a Newtonian incompressible fluid. We used a penalty method which is based on a reformulation of the stress tensor which allows the canceling of the deformation rate in the volume occupied by the particle. This method consists on constraining the movement of the fluid to be the same as the movement of a particle by increasing locally the viscosity of the fluid [10, 11, 12]. This method has been used by many authors, initially to consider the Dirichlet condition at the edge of the field, and then to deal with the presence of an obstacle within a flow. It has been expanded recently to deal with the stress of rigid motion for a particle in a fluid for a finite differences approach then for finite elements [13, 14].

The objective of this work is to develop a code from FreeFem ++ that simulates Stokes or Navier-Stokes flows (with low Reynolds number) in the presence of solid particles. A test case on the sedimentation of a particle is presented.

2 Mathematical formulation of the problem

We consider a connected, bounded and regular domain $\Omega \subset \mathbb{R}^2$ (see Fig.1) and we denote by $(B_i)_{i=1,\cdots,N}$ the rigid particles, strongly included in Ω. B denotes the whole rigid domain: $B = \cup_i B_i$. The domain $\Omega \setminus \overline{B}$ is filled with Newtonian fluid governed by the Navier-Stokes equations. We note μ the viscosity of the fluid, p the presure and $\mathrm{f_f}$ the external forces exerted on it. Since we consider a Newtonian fluid, the stress tensor $\underline{\underline{\sigma}}$ is given by the following relation (see Eq. (1)):

$$\underline{\underline{\sigma}} = 2\mu\mathbb{D}(\mathbf{u}) - p\mathbb{I}, \quad \text{where} \quad \mathbb{D}(\mathbf{u}) = \frac{\nabla(\mathbf{u}) + (\nabla(\mathbf{u}))^T}{2} \tag{1}$$

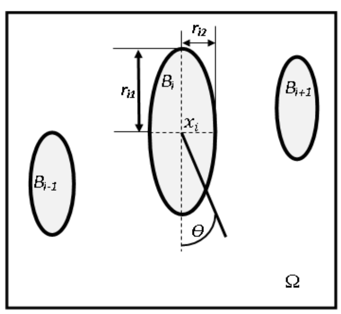

Figure 1: Partcicles B_i in a Newtonian fluid.

We consider homogeneous Dirichlet conditions on $\partial\Omega$. The presence of viscosity imposes a no-slip condition on the boundary ∂B of the rigid domain.
At the initial time the particles with density ρ_i are distributed randomly over the fluid. The position of the center of the ith particle is denoted by x_i, by v_i and ω_i its translational and angular velocities. We denote by m_i and J_i the mass and the kinematic momentum about its center of mass:

$$m_i = \int_{B_i} \rho_i, \quad J_i = \int_{B_i} \rho_i \|x - x_i\|^2 \tag{2}$$

We have to find the velocity $\mathrm{u}(u_1, u_2)$ and the pressure field p defined in $\Omega \setminus \overline{B}$, as well as the velocities of the particles$\mathbf{V} := (\mathrm{v}_{\mathrm{i}=1,\ldots,\mathrm{N}}) \in \mathbb{R}^{2N}$ and $\boldsymbol{\omega} := (\omega_{\mathrm{i}=1,\ldots,\mathrm{N}}) \in \mathbb{R}^N$ such that (see Eq. (3)):

$$\begin{cases} \rho_{\mathrm{f}}(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}) - \mathrm{div}(\underline{\sigma}) = \mathbf{f}_{\mathrm{f}} & \text{dans } \Omega \setminus \overline{B}, \\ \mathrm{div}(\mathbf{u}) = 0 & \text{in } \Omega \setminus \overline{B}, \\ \mathbf{u} = 0 & \text{on } \partial\Omega, \\ \mathbf{u} = \mathbf{v}_i + \omega_i (x - x_i)^{\perp} & \text{on } \partial B,\ \forall i \in \{1, \ldots, N\} \end{cases} \tag{3}$$

where ρ_{f} denotes the density of the fluid and $\mathbf{f}_{\mathrm{f}} = \rho_{\mathrm{f}} g e_y$ is the external force exerted on the fluid (gravity forces). The fluid exerts hydrodynamic forces on the particles. Newton's second law for these particles is written then as follows (see Eq. (4)):

$$\begin{cases} m_i \frac{dV_i}{dt} = \int_{B_i} \mathrm{f_i} - \int_{\partial B_i} \underline{\sigma} n, \\ J_i \frac{d\omega_i}{dt} = \int_{B_i} (x - x_i)^{\perp}.\mathrm{f_i} - \int_{\partial B_i} (x - x_i)^{\perp}.\underline{\sigma} n, \end{cases} \tag{4}$$

Where, $\mathrm{f_i}$ denotes the external non-hydrodynamical forces exerted on the sphere, such as gravity : $\mathrm{f_i} = -\rho_i g e_y$.

3 Variational formulation and Penalisation method

The variational formulation obtained on the whole fluid/particle domain Ω is given here after (see Eq. (5)):

$$\begin{cases} Find\ (\mathbf{u}, p) \in \mathbf{K}_B \times \mathbf{L}_0^2(\Omega)\ such that \\ \int_\Omega \tilde{\rho} \frac{D\mathbf{u}}{Dt} \mathbf{v} + 2\mu \int_\Omega \mathbb{D}(\mathbf{u}) : \mathbb{D}(\mathbf{v}) - \int_\Omega p \mathrm{div}(\mathbf{v}) = \int_\Omega \tilde{\mathrm{f}}.\mathbf{v},\ \forall \mathbf{v} \in \mathbf{K}_B \\ \int_\Omega q \mathrm{div}(\mathbf{u}) = 0,\ \forall q \in \mathbf{L}_0^2, \end{cases} \tag{5}$$

with $\tilde{\rho} := \rho_f \mathbf{1}_{\Omega\setminus\overline{B}} + \sum_{i=1}^N \rho_f \mathbf{1}_{B_i}$, $\quad \tilde{\mathrm{f}} := \mathrm{f}_f \mathbf{1}_{\Omega\setminus\overline{B}} + \sum_{i=1}^N \mathrm{f}_i \mathbf{1}_{B_i}\, and\, \mathbf{K}_B = \{\mathbf{u} \in H_0^1(\Omega) \mathbb{D}(\mathbf{u}) = 0\, in B\}$.
Using a penalty method, we will rather consider the following problem:

$$
\begin{cases}
Find\,(\mathbf{u},p)\in\mathbf{H}_0^1(\Omega)\times\mathbf{L}^2(\Omega)\,such that\\
\int_\Omega\tilde{\rho}\,\frac{D\mathbf{u}}{Dt}\mathbf{v}+2\mu\int_\Omega\mathbb{D}(\mathbf{u}):\mathbb{D}(\mathbf{v})+\frac{2}{\varepsilon}\int_B\mathbb{D}(\mathbf{u}):\mathbb{D}(\mathbf{v})\\
\qquad\qquad -\int_\Omega p\mathrm{div}(\mathbf{v})=\int_\Omega\tilde{\mathrm{f}}.\mathbf{v},\ \forall\mathbf{v}\in\mathbf{H}_0^1(\Omega),\\
\qquad\qquad \int_\Omega q\mathrm{div}(\mathbf{u})=0,\ \forall q\in\mathbf{L}^2(\Omega),
\end{cases}
\tag{6}
$$

The variational formulation (see Eq. (6)) shows that the physics behind this method is to consider the rigid domain as a fluid with infinite viscosity.
The time discretization is performed by using the method of characteristics [15].

4 Results

In Fig. 2, we show the results of the sedimentation of elliptic particle in a closed box filled with a Navier-Stokes fluid at different time steps.

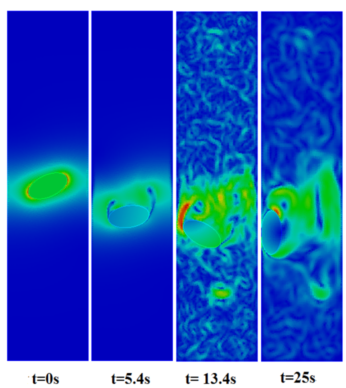

Figure 2: Sedimentation of particule -configurations at different time steps-

5 Conclusion

In this paper, we have proposed a strategy for the numerical modeling of the motion of a convex rigid particle in a Newtonian fluid. The rigid motion is imposed by penalizing the strain tensor, the time discretization is performed by using the method of characteristics.

The code was written in FreeFem++ version 3.26 and at each time step the generalized Navier-Stokes problem is solved by using standard finite elements.

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