Construction of Analytical Solutions to Nonlinear Evolution Equations Using the Generalized Differential Operator Method

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There are a number of mathematical tools which, with the help of computer algebra, provide powerful techniques for the solution of nonlinear evolutions in mathematical physics. The generalized differential operator method, first introduced in [1], has been successfully applied for the construction of solitary solutions to equations in mathematical physics [2, 3]. In this talk, we consider the use of this method to construct more general closed-form analytical solutions.

Since the wave variable substitution $x = \xi - at$ transforms nonlinear evolution equations into ordinary differential equations, we consider the following initial value problem:

$$y''_{xx} = P\left(x, y, y'_x\right); \quad y = y\left(x; c, s, t\right), \quad y\left(c; c, s, t\right) = s, \quad y'_x\left(c; c, s, t\right) = t. \qquad (1)$$

The generalized differential operator for (1) reads $\mathrm{D} := \mathrm{D}_c + t\mathrm{D}_s + P(c,s,t)\mathrm{D}_t$. Denoting $p_j(c,s,t) := \mathrm{D}^j s$, the general solution to (1) can be expressed as the series $y = y(x;c,s,t) = \sum_{j=1}^{+\infty} \frac{(x-c)^j}{j!} p_j(c,s,t)$. We show that if $f(x) = \sum_{j=1}^{+\infty} \frac{q_j}{j!} x^j$ is an arbitrary analytical function and the sequence $\widehat{p}_j = \frac{p_j}{q_j}$ is a linear recurring sequence of order n [4], the solution to (1) can be written in closed form as $y = \sum_{k=1}^{n} \lambda_k f\left(\alpha_k - \beta_k \left(x-c\right)\right)$, where $\lambda_k, \alpha_k, \beta_k \in R$ are constants.

The outlined algorithm is a powerful tool for the construction of closed-form solutions to nonlinear evolution equations in mathematical physics.

References

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