Symbolic Contour Integration in Mathematica (Part 2) :
Some Special Topics to be Investigated

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While the first part was devoted only to the main procedures calculateResidues and ContourIntegration applied to a wide class of complex functions $f(z)$ which are rational polynomials, products of rational and trigonometric/hyperbolic functions, rational functions consisting of trigonometric/hyperbolic functions. However, the investigations of the second part of this paper are special topics which occur in the context of contour integration and are of interest in itselves. The issues discussed in this paper are :
(1) introduction of a language for creation and visualization of non-trivial integration paths consisting of polylines and circular arcs such as contours γ which exclude certain poles or branch cuts, or the sophisticated contour for Meijer G-functions meandering around integer singularities but avoiding half-integral ones, or the Pochhammer double-loop contour for the evaluation of the so-called Euler's integral etc. ;
(2) criterium for the determination of poles inside/outside an arbitrary closed contour ;
(3) visualization and extraction of (non-trivial) branch cuts for various functions $f(z)$ such as $\sqrt{\sin(z)}$ or $\arcsin(z^n)$;
(4) treatment of contour integrals for which the integrands possess branch cuts such as $f(z) = \frac{\sqrt{z}}{z^2+1}$;
(5) transformation of improper integrals (along the real axis) into exotic contour integrals with the help of change of variables, e.g. $\int_0^{+\infty} \frac{1}{x^3+1}\, dx$ with variables $x \to r e^{i\,\phi}$ where $\phi \to \frac{2\pi}{3}$ etc. ;
(6) symbolic evaluation of the integral representation for special functions such as Meijer G-function or Euler's integral for Beta function .

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